**Finding square roots by guess & check method**

To find a decimal approximation to, say √2, first make an initial guess, then square the guess, and depending how close you got, improve your guess. Since this method involves squaring the guess (multiplying the number times itself), **it uses the actual definition of square root**, and so can be very helpful in teaching the concept of square root.

**Example: what is square root of 20?**

You can start out by noting that since √16 = 4 and √25 = 5, then √20 must be between 4 and 5.

Then make a guess for √20; let's say for example that it is 4.5. Square that, see if the result is over or under 20, and improve your guess based on that. Repeat this process until you have the desired accuracy (amount of decimals). It's that simple and can be a nice experiment for students!

**Example: Find √6 to 4 decimal places**

Since 22 = 4 and 32 = 9, we know that √6 is between 2 and 3. Let's guess (or estimate) that it is 2.5. Squaring that we get 2.52 = 6.25. That's too high, so we reduce our estimate a little. Let's try 2.4 next. To find the square root of 6 to four decimal places we need to repeat this process until we have five decimals, and then we will round the result.

|  |  |  |
| --- | --- | --- |
| Estimate | Square of estimate | High/low |
| 2.4 | 5.76 | Too low |
| 2.45 | 6.0025 | Too high but real close |
| 2.449 | 5.997601 | Too low |
| 2.4495 | 6.00005025 | Too high so the square root of 6 must be between 2.449 and 2.4495. |
| 2.4493 | 5.99907049 | Too low |
| 2.4494 | 5.99956036 | Too low, so the square root of 6 must be between 2.4494 and 2.4495 |
| 2.44945 | 5.9998053025 | Too low, so the square root of 6 must be between 2.44945 and 2.4495. |

This is enough iterations since we know now that √6 would be rounded to 2.4495 (and not to 2.4494).

**Finding square roots using an algorithm**

There is also an algorithm for square roots that resembles the long division algorithm, and it was taught in schools in days before calculators. See the example below to learn it. While learning this algorithm may not be necessary in today's world with calculators, working out some examples can be used as an exercise in basic operations for middle school students, and [**studying the logic behind it**](https://www.homeschoolmath.net/teaching/sqr-algorithm-why-works.php) can be a good thinking exercise for high school students.

**Example: Find √645 to one decimal place.**

First group the numbers under the root in pairs from right to left, leaving either one or two digits on the left (6 in this case). For each pair of numbers you will get one digit in the square root.

To start, find a number whose square is less than or equal to the first pair or first number, and write it above the square root line (2):

|  |  |
| --- | --- |
| 2 |  |
| **√**6 | .45 |

Then continue this way:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|

|  |  |
| --- | --- |
| 2 |  |
| **√**6 | .45 |
|  - 4 |  |
| 2 | 45 |

 |

|  |  |  |
| --- | --- | --- |
|  | 2 |  |
|  | **√**6 | .45 |
|  |  - 4 |  |
| (4 \_) | 2 | 45 |

 |

|  |  |  |
| --- | --- | --- |
|  | 2 |  |
|  | **√**6 | .45 |
|  |  - 4 |  |
| (45) | 2 | 45 |

 |
| Square the 2, giving 4, write that underneath the 6, and subtract. Bring down the next pair of digits. | Then double the number above the square root symbol line (highlighted), and write it down in parenthesis with an empty line next to it as shown. | Next think what single-digit number *something*could go on the empty line so that forty-*something* times *something*would be less than or equal to 245.45 x 5 = 22546 x 6 = 276, so 5 works. |
|

|  |  |  |  |
| --- | --- | --- | --- |
|  | 2 | 5 |  |
|  | **√**6 | .45 | **.**00 |
|  |  - 4 |  |  |
| (45) | 2 | 45 |  |
|  | - 2 | 25 |  |
|  |  | 20 | 00 |

 |

|  |  |  |  |
| --- | --- | --- | --- |
|  | 2 | 5 |  |
|  | **√**6 | .45 | **.**00 |
|  |  - 4 |  |  |
| (45) | 2 | 45 |  |
|  | - 2 | 25 |  |
| (50\_) | 20 | 00 |
|  |  |  |  |

 |

|  |  |  |  |
| --- | --- | --- | --- |
|  | 2 | 5 | **.** 3 |
|  | **√**6 | .45 | **.**00 |
|  |  - 4 |  |  |
| (45) | 2 | 45 |  |
|  | - 2 | 25 |  |
| (503) | 20 | 00 |
|  |  |  |  |

 |
| Write 5 on top of line. Calculate 5 x 45, write that below 245, subtract, bring down the next pair of digits (in this case the decimal digits 00). | Then double the number above the line (25), and write the doubled number (50) in parenthesis with an empty line next to it as indicated: | Think what single digit number *something* could go on the empty line so that five hundred-*something* times *something*would be less than or equal to 2000. 503 x 3 = 1509504 x 4 = 2016, so 3 works. |
|

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 2 | 5 | **.** 3 |  |
|  | **√**6 | .45 | **.**00 | .00 |
|  |  - 4 |  |  |  |
| (45) | 2 | 45 |  |  |
|  | - 2 | 25 |  |  |
| (503) | 20 | 00 |  |
|  | -  | 15 | 09 |  |
|  |  | 4 | 91 | 00 |

 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 2 | 5 | **.** 3 |  |
|  | **√**6 | .45 | **.**00 | .00 |
|  |  - 4 |  |  |  |
| (45) | 2 | 45 |  |  |
|  | - 2 | 25 |  |  |
| (503) | 20 | 00 |  |
|  | -  | 15 | 09 |  |
| (506\_)  | 4 | 91 | 00 |

 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 2 | 5 | **.** 3 | 9 |
|  | **√**6 | .45 | **.**00 | .00 |
|  |  - 4 |  |  |  |
| (45) | 2 | 45 |  |  |
|  | - 2 | 25 |  |  |
| (503) | 20 | 00 |  |
|  | -  | 15 | 09 |  |
| (506\_)  | 4 | 91 | 00 |

 |
| Calculate 3 x 503, write that below 2000, subtract, bring down the next digits. | Then double the 'number' 253 which is above the line (ignoring the decimal point), and write the doubled number 506 in parenthesis with an empty line next to it as indicated: | 5068 x 8 = 405445069 x 9 = 45621, which is less than 49100, so 9 works. |

Thus to one decimal place, **√**645 = 25.4